
Student number

2024 **TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced

General instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of this page

Total marks: 100

Section I – 10 marks (pages 2–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8–32)

- Attempt Questions 11–35
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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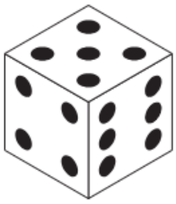
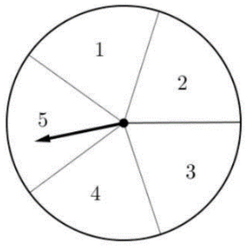
1 A game involves throwing a standard die and spinning a spinner numbered 1 to 5.





The product of the two numbers is the score.

The table of scores below is partially completed.

		SPINNER				
		1	2	3	4	5
DIE	1	1	2	3	4	
	2	2	4	6		
	3	3	6			
	4	4				
	5					
	6					

What is the probability of obtaining an even score?

A. $\frac{3}{10}$

B. $\frac{5}{10}$

C. $\frac{6}{10}$

D. $\frac{7}{10}$

2 An athlete runs for 500 metres around a circular track with radius 200 metres.

How far, in a direct line across the field, is the athlete from her starting position?

A. 200 m

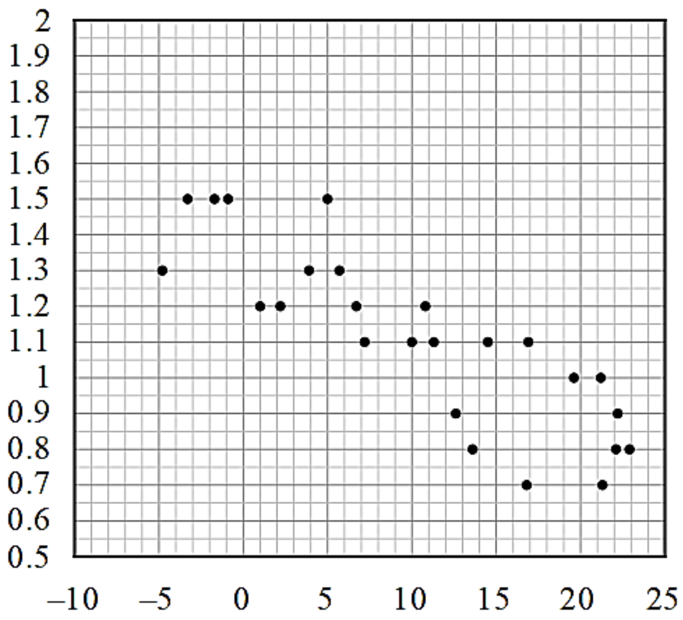
B. 283 m

C. 380 m

D. 458 m

3 A researcher assumes that there is a linear relationship between the temperature and hot chocolate consumption.

Data was collected randomly in cities across Europe and the results were recorded in the scatterplot below.



The scatterplot shows the average monthly hot chocolate consumption, in litres/person against average monthly temperature, in °C.

The correlation coefficient for this data would be closest to:

A. –0.8

B. –0.4

C. 0.4

D. 0.8

4 A set of scores in a data set has a median of 72, an interquartile range of 14 and a variance of 4.

Each score has 3 added to it and each resulting score is then doubled to form a new set of scores.

What is the median, interquartile range, and variance of the new set of scores?

	Median	Interquartile range	Variance
A.	144	28	8
B.	150	28	16
C.	150	32	8
D.	150	32	16

5 How many solutions does $\sec\theta + \tan\theta = 0$ have for $-\pi \le \theta \le \pi$?

A. 0

B. 1

C. 2

D. 3

6 Let $g(x) = \sqrt{2-x}$. What is the domain of the composite function $g(g(x))$?

A. $x \le \sqrt{2}$

B. $x \le 2$

C. $-\sqrt{2} \le x \le \sqrt{2}$

D. $-2 \le x \le 2$

7 Which of the following shows the graph of $f(x)=(x+a)^2(b-x)$, if $a>0$ and $b>0$?

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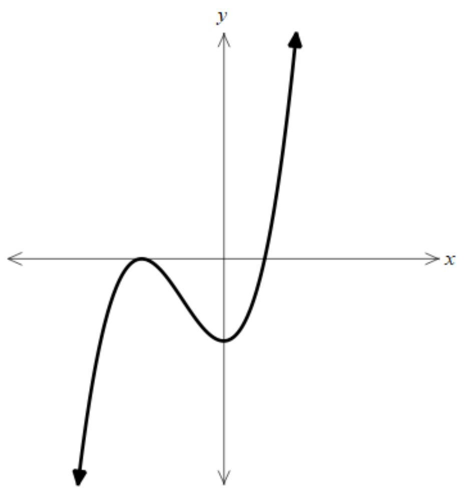
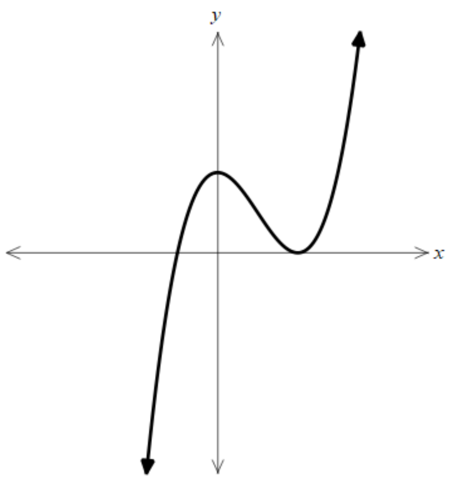
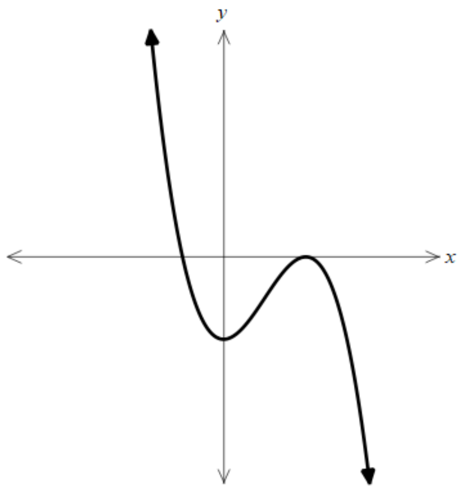


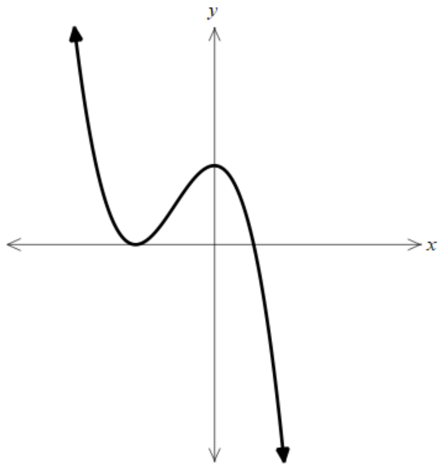
B.



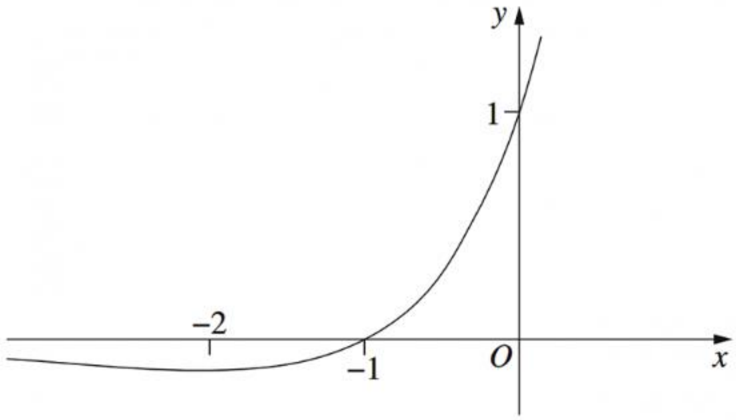
C.



D.



8 The diagram shows the graph of $y = e^x(1+x)$.



How many solutions are there to the equation $e^x(1+x) = 1-(x+1)^2$?

A. 0

B. 1

C. 2

D. 3

9 Given that $\int_1^6 f(x)\,dx = -2$ and $\int_3^6 f(x)\,dx = 6$, what is the value of $\int_1^3 \left(f(x)+x\right)dx$?

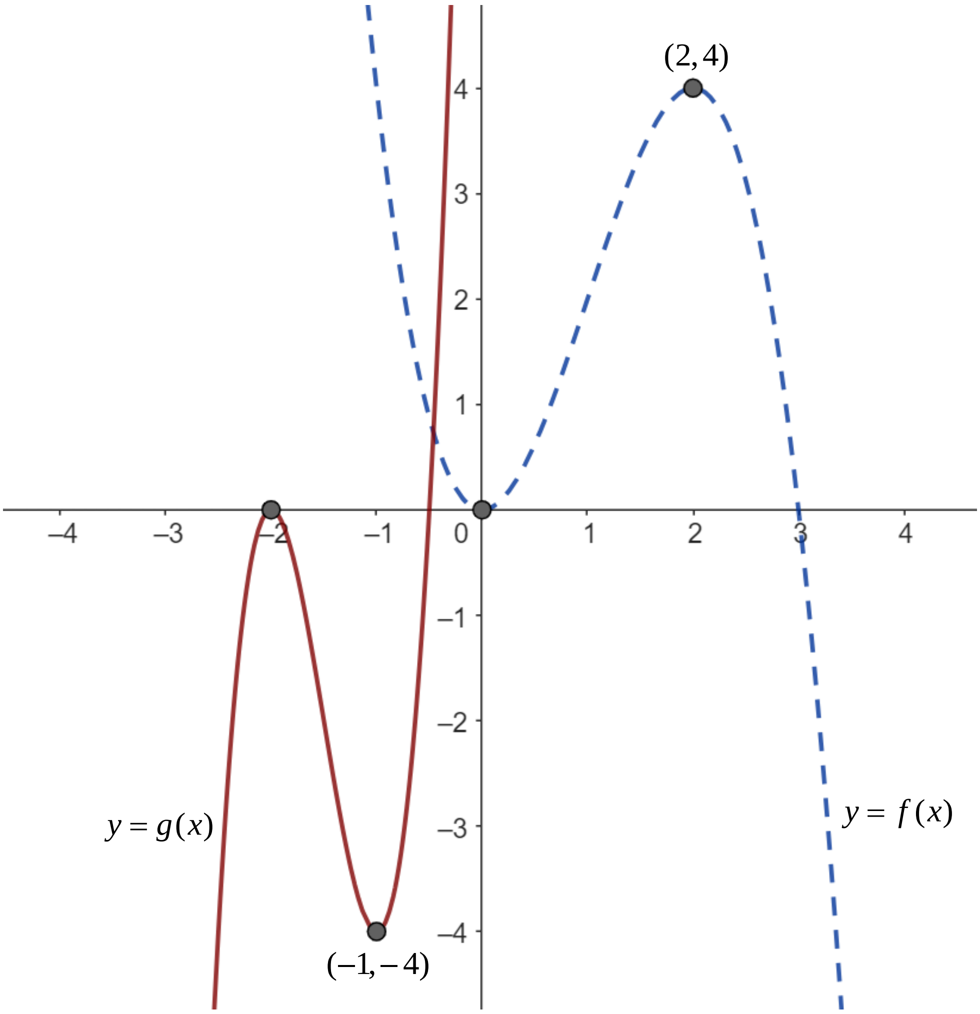
A. –8

B. –4

C. 4

D. 8

10 The graphs of $y = f(x)$ and $y = g(x)$ are shown. The function $y = g(x)$ can be obtained by applying a series of transformations to $y = f(x)$.



Which of the following is correct?

A. $g(x) = -f(x+4)$

B. $g(x) = -f(2x+4)$

C. $g(x) = f(-2x) - 4$

D. $g(x) = f(-x+2) - 4$

Student number

Mathematics Advanced
Section II Answer booklet A

50 marks
Attempt Questions 11–26
Allow about 1 hour and 30 minutes for this section

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

ABC is a triangle in which $\cos A = -\frac{2}{5}$. Find the exact value of $\tan A$. **2**

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Question 12 (2 marks)

Solve $|3x+2| = 4$ **2**

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Question 13 (3 marks)

Find the exact equation of the tangent to the curve $y = \log_2 x$ at the point where $x = 4$. **3**

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Question 14 (4 marks)

Barry owns an ice cream cart. The table below shows the number of ice creams Barry sold in a one hour period, on eight different days whose temperatures were measured in degrees Celsius.

Temperature	18	21	23	26	27	29	32	34
Ice creams sold	2	4	9	16	16	25	34	37

a) Find the equation of the least-squares line of best fit for this data, rounding your answers to 2 decimal places. **2**

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b) Use your equation to predict how many ice creams Barry would sell in one hour when the temperature is 30°C. **1**

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c) Explain why the value of the y intercept for the line of best fit has no meaning in this context. **1**

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Question 15 (2 marks)

Differentiate $\dfrac{(2x+1)^2}{4+x}$ with respect to x **2**

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Question 16 (3 marks)

a) Show that $\dfrac{d}{dx}\left(x^2e^{x^2}\right) = 2xe^{x^2}\left(1+x^2\right)$. **2**

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b) Hence show that $y = x^2e^{x^2}$ is decreasing for all $x < 0$. **1**

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Question 17 (2 marks)

Find $\int \frac{x-3}{x^2-6x}\,dx$ **2**

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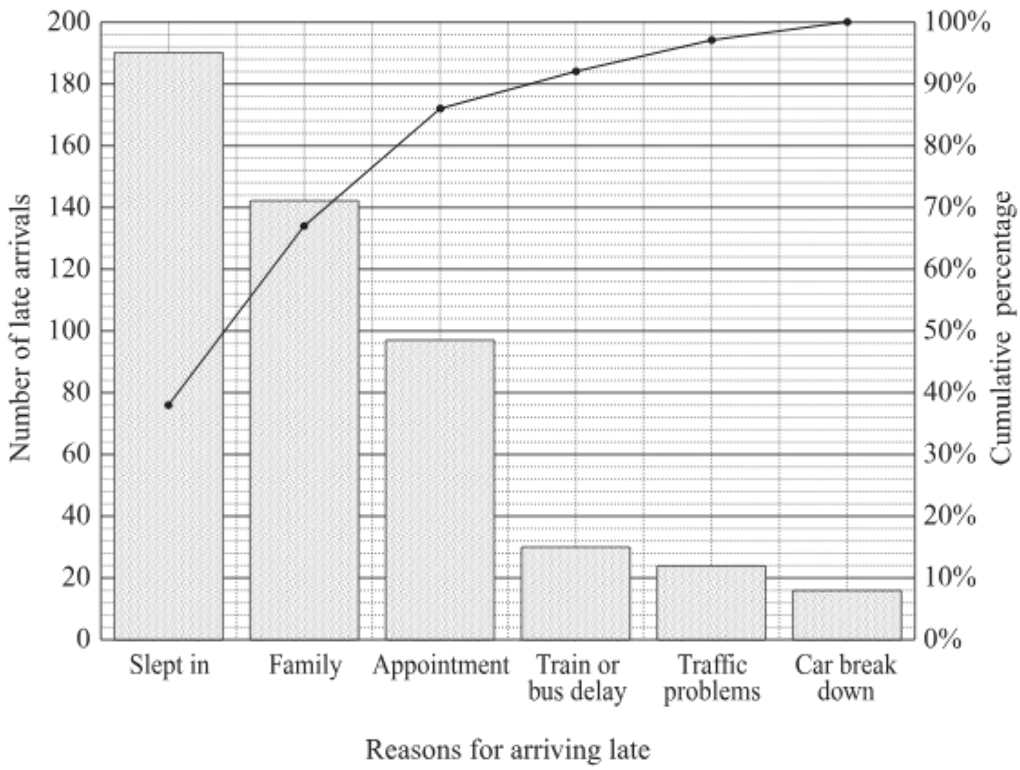
Question 18 (1 mark)

A school collected data on the reasons given by students for arriving late. The data is displayed in the Pareto chart below. **1**



What percentage of students gave the reasons traffic problems or car break down?

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Question 19 (4 marks)

A bag of potatoes is labelled as having a net weight of 2 kilograms. The packaging process produces bags of potatoes whose weights are normally distributed with a mean of 2.05 kilograms and a standard deviation of 0.05 kilograms.

a) In a supermarket selling 2000 of these bags of potatoes, how many bags will contain less than the labelled weight? **2**

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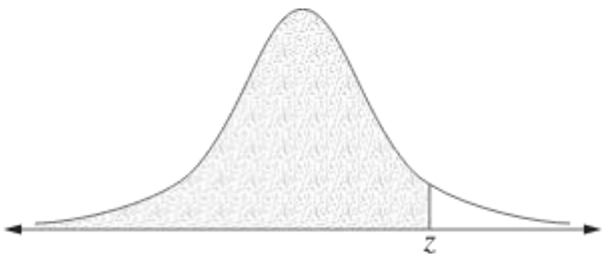
b) For a random variable which is normally distributed with a mean of 0 and a standard deviation of 1, the table gives the probability that this random variable lies below z for some positive values of z. **2**

z	0.80	0.81	0.82	0.83	0.84	0.85	0.86	0.87
Probability	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078

The probability values given in the table are represented by the shaded area in the following diagram.



Using this table, above what weight would the heaviest 20% of bags of potatoes weigh?

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Question 20 (5 marks)

Sixty people visited a swimming pool one afternoon. Thirteen out of the twenty-three people who wore goggles were adults. There were nineteen children.

a) Complete the two-way table below, showing this information. **2**

	Wore goggles	No goggles	Total
Adults			
Children			
Total			

b) If an adult is selected at random, what is the probability they were wearing goggles? **1**

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c) An additional group of adults, not wearing goggles, arrived at the pool. **2**
The probability that a person selected at random is an adult who is not wearing goggles is now 0.6.

How many additional adults arrived?

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Question 21 (8 marks)

The derivative of a function $y = f(x)$ is given by $f'(x) = 3x^2 - 4x + 1$

a) Find the x-coordinates of the stationary points of $y = f(x)$ and determine the nature of the stationary points. **3**

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b) The curve $y = f(x)$ has a y-intercept at $y = -2$. **2**

Find an expression for $f(x)$.

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c) Sketch the curve, clearly indicating the stationary points and intercepts on the axes. **2**

d) For what values of x is the curve concave down? **1**

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Question 22 (3 marks)

A and B are two events such that $P(A) = 0.3$, $P(B) = 0.5$ and $P(A|B) = 0.4$.

a) Show that $P(A \cup B) = 0.6$ **2**

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b) Find P(neither A nor B) **1**

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Question 23 (3 marks)

Find two possible equations of a semi-circle with range $[3,5]$, given that it is an even function. **3**

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Question 24 (2 marks)

A lift accelerates from rest at a constant rate until it reaches a speed of 5 metres per second. It continues at this speed for 12 seconds and then decelerates at a constant rate before coming to rest. The total travel time for the lift is 30 seconds. **2**

Find the total distance, in metres, travelled by the lift.

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Question 25 (3 marks)

The table shows the probability distribution of a discrete random variable.

x	0	1	2	3	4
$P(X=x)$	p^2+p	p^2	p	$2p^2$	p

a) Show that $p=\frac{1}{4}$. **2**

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b) Find the expected value $E(X)$ **1**

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Question 26 (3 marks)

Show that $\log_e(1+\sin x)-\log_e(1-\sin x)=2\log_e(\sec x+\tan x)$ **3**

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Student number

Mathematics Advanced
Section II Answer booklet B

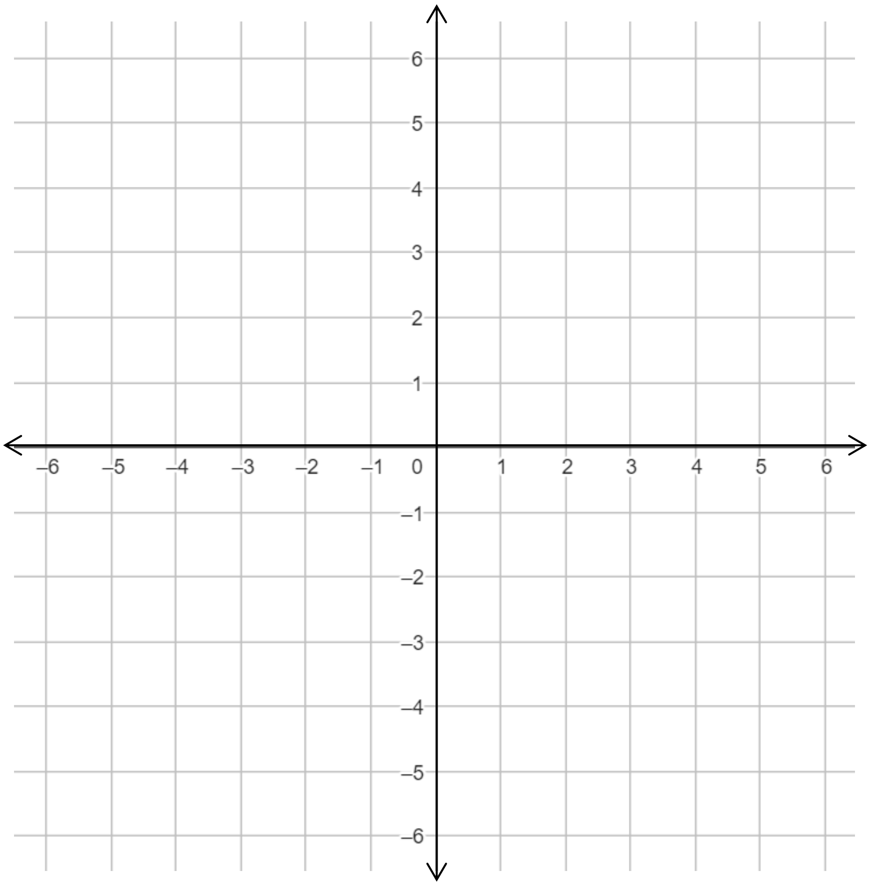
40 marks
Attempt Questions 27–35
Allow about 1 hour and 15 minutes for this section

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 27 (6 marks)

a) Sketch the graphs of the functions $y = \frac{4}{x}$ and $y = |x+1| - 4$ on the number plane below. Clearly show their point(s) of intersection and any intercepts on the axes. **4**



b) Hence, or otherwise, solve $|x+1| - 4 \le \frac{4}{x}$ **2**

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Question 28 (5 marks)

Consider $f(x) = \sqrt{1 - x^2}$

a) Use the trapezoidal rule with 4 subintervals to estimate $\int_0^1 \sqrt{1 - x^2}\, dx$. **2**

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b) Find the exact value of $\int_0^1 \sqrt{1 - x^2}\, dx$ and hence find an approximation for π correct to 3 decimal places. **2**

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c) Explain why this value under-estimates the exact value of π. **1**

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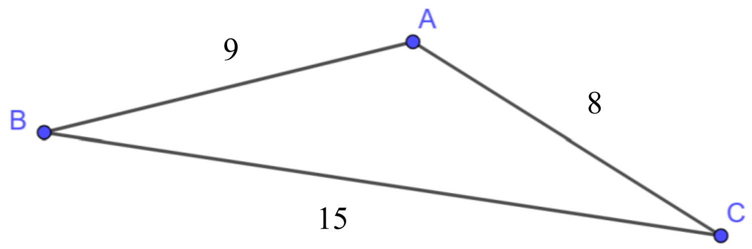
Question 29 (4 marks)

The diagram shows a triangle ABC where $AB = 9$, $AC = 8$ and $BC = 15$. **4**



Find the area of the triangle ABC. Give your answer correct to one decimal place.

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Question 30 (4 marks)

a) Show that $\dfrac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} \equiv \sec^2 x$. **2**

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b) Hence, or otherwise, find the exact value of $\int_0^{\frac{\pi}{3}} \tan^2 x \left(\dfrac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} \right) dx$ **2**

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Question 31 (1 mark)

For a given discrete probability distribution, $E(X)=5$ and $\text{Var}(X)=3$. **1**

Find the value of $E(2X+1)$.

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Question 32 (5 marks)

A wire of length L is bent to form a sector with radius r and angle θ.

a) Show that $\theta=\frac{L-2r}{r}$. **2**

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b) Show that the area A of the sector is maximised when $r=\frac{L}{4}$. **3**

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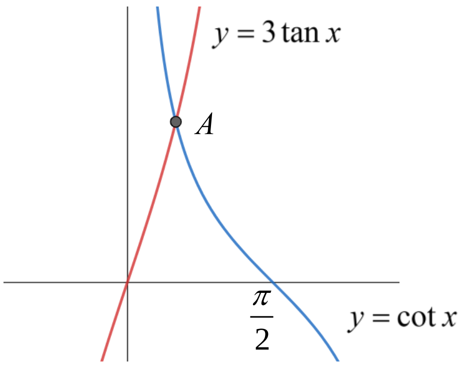
Question 33 (5 marks)

Part of each of the curves $y = 3\tan x$ and $y = \cot x$ are shown on the graph below.

The curve $y = \cot x$ cuts the x axis at $x = \dfrac{\pi}{2}$ and the two curves intersect at the point A.



a) Show that the x coordinate of A is $x = \dfrac{\pi}{6}$. **2**

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b) By considering $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$, find the exact area enclosed by the curves and the x axis between $x = 0$ and $x = \frac{\pi}{2}$. **3**

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Question 34 (2 marks)

A farming company harvests oranges to sell to grocery stores. Oranges that weigh less than 150 grams are rejected and used for juicing instead. Typically, 16% of all oranges are rejected. Also, oranges that are larger than 300 grams are reserved for restaurants. Typically, 2.5% of all oranges are sold to restaurants. **2**

If the weights of the oranges are normally distributed, find the mean and standard deviation of the normal distribution.

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Question 35 (8 marks)

A continuous random variable X has the probability density function $f(x)$ given by

$$f(x) = \begin{cases} k(\sin(\pi x) + 2x), & \text{for } 0 \le x \le 3 \\ 0, & \text{for all other values of } x \end{cases}$$

a) Show that $k = \dfrac{\pi}{2 + 9\pi}$. **3**

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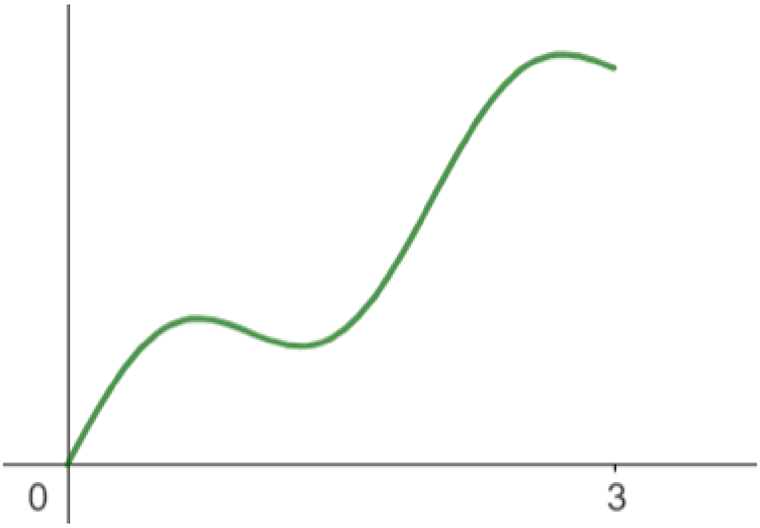
b) The graph of the density function is given below. **1**



Without any calculation, explain why the mode must be greater than the median.

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c) Find the mode of X. Give your answer correct to two decimal places. **4**

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Section I

Multiple choice Answer Key

Question	Answer
1	D
2	C
3	A
4	B
5	A
6	D
7	D
8	C
9	B
10	B

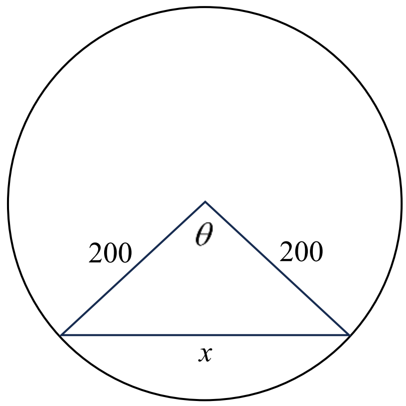
Question 1

		SPINNER				
		1	2	3	4	5
DIE	1	1	②	3	④	5
	2	②	④	⑥	⑧	⑩
	3	3	⑥	9	⑫	15
	4	④	⑧	⑫	⑯	⑳
	5	5	⑩	15	⑳	25
	6	⑥	⑫	⑱	(24)	(30)

$$P(\text{even}) = \frac{21}{30}$$

$$= \frac{7}{10} \quad \Rightarrow D$$

Question 2



$$l = r\theta$$
$$500 = 200\theta$$
$$\theta = 2.5$$

$$x^2 = 200^2 + 200^2 - 2\times 200\times 200\times \cos 2.5$$
$$x^2 = 144091.4892...$$
$$x = 379.59... \qquad \Rightarrow C$$

Question 3

Strong, negative correlation

$\therefore -1 < r < -0.7 \qquad \Rightarrow A$

Question 4

Median is a specific score so it also has 3 added and then is doubled. Interquartile range and variance are measures of spread, so are unaffected by adding 3 to each score.

$$\text{Median} = (72+3)\times 2$$
$$= 150$$
$$\text{IQR} = 14\times 2$$
$$= 28$$
$$\text{Variance} = 4\times 2^2$$
$$= 16 \qquad \Rightarrow B$$

Question 5

$$\sec\theta + \tan\theta = 0\ , \quad -\pi \le \theta \le \pi$$
$$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = 0$$
$$1 + \sin\theta = 0\ , \quad \cos\theta \ne 0$$
$$\sin\theta = -1$$
$$\theta = -\frac{\pi}{2}$$

But $\cos\theta \ne 0$, $\therefore \theta \ne -\frac{\pi}{2}$

$\therefore$ no solutions $\qquad \Rightarrow A$

Question 6

$g(g(x)) = \sqrt{2-\sqrt{2-x}}$

$2 - x \geq 0$

$x \leq 2$

$2 - \sqrt{2-x} \geq 0$

$\sqrt{2-x} \leq 2$

$0 \leq 2 - x \leq 4$

$-2 \leq -x \leq 2$

$-2 \leq x \leq 2 \qquad \Rightarrow D$

Question 7

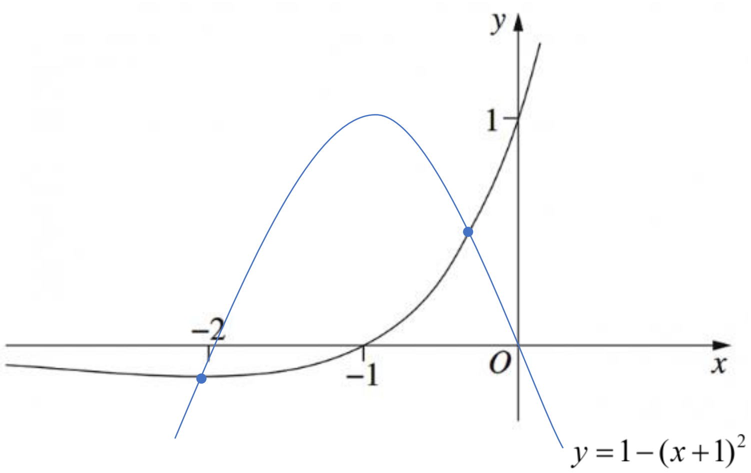
$f(x) = (x+a)^2(b-x)$

Double root at $x = -a$

Single root at $x = b$

As $x \to \infty$, $y \to -\infty \qquad \Rightarrow D$

Question 8



Two points of intersection between graphs $\qquad \Rightarrow C$

Question 9

$$\int_1^3 \left(f(x) + x \right) dx = \int_1^3 f(x)\, dx + \int_1^3 x\, dx$$
$$= \int_1^6 f(x)\, dx - \int_6^6 f(x)\, dx + \left[\frac{x^2}{2} \right]_1^3$$
$$= -2 - 6 + \left[\frac{9}{2} - \frac{1}{2} \right]$$
$$= -4 \qquad \Rightarrow B$$

Question 10

The graph of $y = g(x)$ can be obtained from $y = f(x)$ by applying:

A horizontal dilation by a factor of $\frac{1}{2} \rightarrow f(2x)$

Translation by 2 units to left $\rightarrow f\left(2(x+2)\right)$

Reflection on *x*-axis: $\rightarrow -f\left(2(x+2)\right)$

$g(x) = -f(2x+4) \qquad \Rightarrow B$

Section II

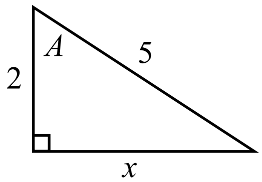
Question 11 (2 marks)

ABC is a triangle in which $\cos A = -\frac{2}{5}$. Find the exact value of $\tan A$.

Sample answer:

$x^2 = 5^2 - 2^2$

$x = \sqrt{21}$



$\cos A < 0 \therefore A$ is obtuse and $\tan A < 0$

$\tan A = -\frac{\sqrt{21}}{2}$

Criteria	Marks
Provides correct solution	**2**
Finds the third side in the triangle, or equivalent merit	**1**
Markers comments	
Mostly well done. Successful responses drew the triangle. Many students incorrectly used Pythagoras' theorem unfortunately. This is from Stage 4 and is assumed knowledge.	

Question 12 (2 marks)

Solve $|3x+2|=4$

Sample answer:

$$3x+2=4 \qquad\qquad 3x+2=-4$$
$$3x=2 \qquad \text{or} \qquad 3x=-6$$
$$x=\frac{2}{3} \qquad\qquad x=-2$$

Criteria	Marks
Provides two correct solutions	**2**
Provides one correct solution, or equivalent merit	**1**

Markers comments
Mostly well done. Many responses only considered one case of the equation.

Question 13 (3 marks)

Find the exact equation of the tangent to the curve $y = \log_2 x$ at the point where $x = 4$.

Sample answer:

$$y' = \frac{1}{(\ln 2)x}$$

$$m_T = \frac{1}{4\ln 2}$$

At $x = 4$, $y = \log_2 4$

$$y = 2$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{4\ln 2}(x - 4)$$

$$y = \frac{x}{4\ln 2} - \frac{1}{\ln 2} + 2$$

Criteria	Marks
Provides correct solution (accept non-simplified forms)	**3**
Finds correct y' AND either correct gradient or y value, or equivalent merit	**2**
Finds correct y' OR correct y value, or equivalent merit	**1**

Markers comments
Mixed results. Many 2's were awarded. This derivative is on your reference sheet!!!! Know and use the reference sheet.

Question 14 (4 marks)

Barry owns an ice cream cart. The table below shows the number of ice creams Barry sold in a one hour period, on eight different days whose temperatures were measured in degrees Celsius.

Temperature	18	21	23	26	27	29	32	34
Ice creams sold	2	4	9	16	16	25	34	37

a) Find the equation of the least-squares line of best fit for this data, rounding your answers to 2 decimal places.

Sample answer:

$A = -44.3975\ldots,\ B = 2.3722\ldots$

$y = 2.37x - 44.40$

Criteria	Marks
Provides correct solution, with correct rounding	**2**
Provides one correct value in the correct position, or Finds the correct values of A and B but around the wrong way, or Finds both correct values but has a rounding error	**1**

Markers comments
Mostly well done. This is straight from your calculator.

b) Use your equation to predict how many ice creams Barry would sell in one hour when the temperature is 30°C.

Sample answer:

$$\begin{aligned} y &= 2.37 \times 30 - 44.40 \\ &= 26.7 \\ &\approx 27 \text{ ice creams} \end{aligned}$$

Criteria	Marks
Provides correct solution	**1**
Markers comments	
Poorly done. Many said 26.7 ice creams sold. You can only sell an integer amount of ice creams. Check that your responses make sense within the context of the question.	

c) Explain why the value of the y intercept for the line of best fit has no meaning in this context.

Sample answer:

Barry can't sell a negative number of ice creams.

Criteria	Marks
Provides a correct answer	**1**
Markers comments	
Poorly done. You cannot use your opinion to interpret data. You must only use the data! You cannot say "because no one buys ice cream in 0 degrees". This was the most common thing amongst the poor responses to this question.	

Question 15 (2 marks)

Differentiate $\dfrac{(2x+1)^2}{4+x}$ with respect to x

Sample answer:

$$u=(2x+1)^2 \qquad v=4+x$$
$$u'=2(2x+1)\times 2 \qquad v'=1$$
$$=8x+4$$

$$\frac{d}{dx}\left[\frac{(2x+1)^2}{4+x}\right]=\frac{(4+x)(8x+4)-(2x+1)^2(1)}{(4+x)^2}$$
$$=\frac{32x+16+8x^2+4x-(4x^2+4x+1)}{(4+x)^2}$$
$$=\frac{4x^2+32x+15}{(4+x)^2}$$

Criteria	Marks
Provides correct solution	**2**
Attempts to use the quotient rule, or equivalent merit	**1**
Markers comments	
Very poorly done. Many incomplete responses. You must simplify. Many concerning algebraic errors were made in poorer responses such as cancelling (4+x) when it is not a common factor.	

Question 16 (3 marks)

a) Show that $\frac{d}{dx}\left(x^2e^{x^2}\right)=2xe^{x^2}\left(1+x^2\right)$.

Sample answer:

$$u = x^2 \qquad v = e^{x^2}$$
$$u' = 2x \qquad v' = 2x\cdot e^{x^2}$$

$$\begin{aligned}\frac{d}{dx}\left(x^2e^{x^2}\right) &= e^{x^2}\times 2x + x^2\times 2xe^{x^2}\\ &= 2xe^{x^2} + 2x^3e^{x^2}\\ &= 2xe^{x^2}\left(1+x^2\right)\end{aligned}$$

Criteria	Marks
Provides correct solution	**2**
Attempts to use the product rule, or equivalent merit	**1**
Markers comments	
This question was well done. Remember with a show that question you must show all necessary steps.	

b) Hence show that $y = x^2 e^{x^2}$ is decreasing for all $x < 0$.

Sample answer:

When $x < 0$, $2x < 0$

$e^{x^2} > 0$

$1 + x^2 > 0$

Thus $y' = \text{negative} \times \text{positive} \times \text{positive}$

$= \text{negative}$

Hence y is decreasing for all $x < 0$.

Criteria	Marks
Provides correct solution	**1**
Markers comments	
Poorly done. You must show/explain why the function is always decreasing for x<0. Showing a case is not showing that this is decreasing.	

Question 17 (2 marks)

Find $\int \frac{x-3}{x^2-6x}\,dx$

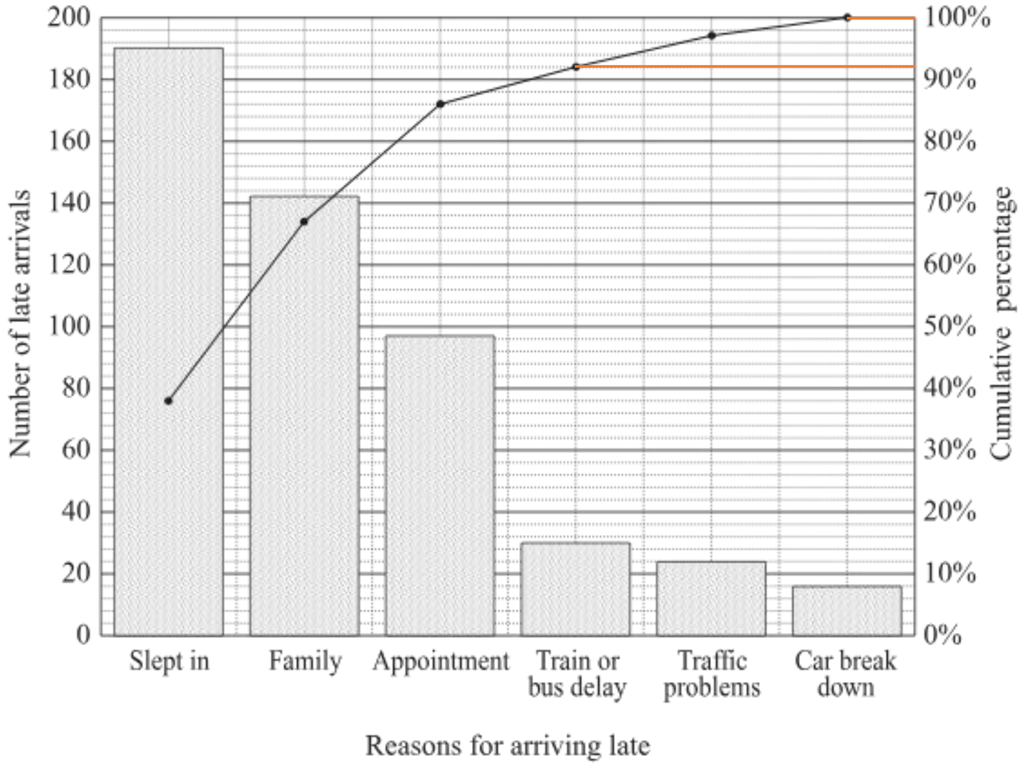
Sample answer:

$$\begin{aligned}\int \frac{x-3}{x^2-6x}\,dx &= \frac{1}{2}\int \frac{2x-6}{x^2-6x}\,dx \\ &= \frac{1}{2}\ln\left|x^2-6x\right|+c\end{aligned}$$

Criteria	Marks
Provides correct solution	**2**
Recognises the integral is of the form $\int \frac{f'(x)}{f(x)}\,dx$, or equivalent merit	**1**
Markers comments	
Well done by the cohort.	

Question 18 (1 mark)

A school collected data on the reasons given by students for arriving late. The data is displayed in the Pareto chart below.



What percentage of students gave the reasons traffic problems or car break down?

Sample answer:

$100 - 92 = 8\%$

Criteria	Marks
Provides correct solution	**1**
Markers comments	
Mostly well done.	

Question 19 (4 marks)

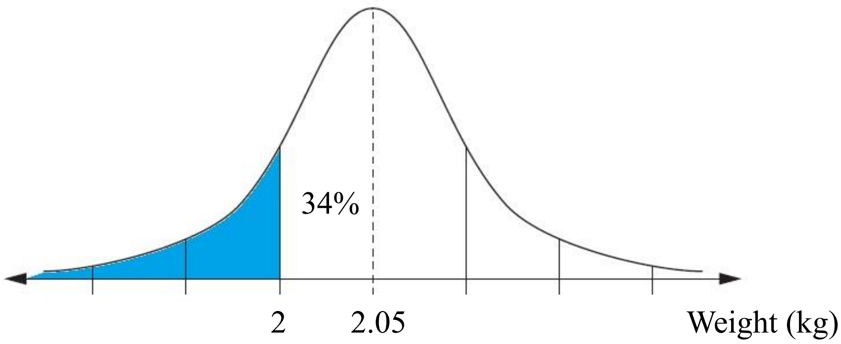
A bag of potatoes is labelled as having a net weight of 2 kilograms. The packaging process produces bags of potatoes whose weights are normally distributed with a mean of 2.05 kilograms and a standard deviation of 0.05 kilograms.

a) In a supermarket selling 2000 of these bags of potatoes, how many bags will contain less than the labelled weight?

Sample answer:

$50 - 34 = 16\%$

$16\% \times 2000 = 320$



Criteria	Marks
Provides correct solution	**2**
Provides correct percentage OEM	**1**

Markers comments

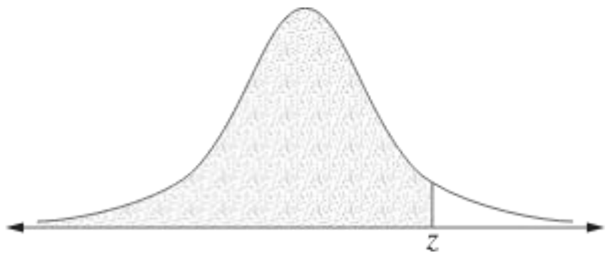
Students largely did this question well. Drawn graphs of the normal curve often assisted in understanding.

Many students who found 16% then did not find the number of bags of potatoes.

b) For a random variable which is normally distributed with a mean of 0 and a standard deviation of 1, the table gives the probability that this random variable lies below z for some positive values of z.

z	0.80	0.81	0.82	0.83	0.84	0.85	0.86	0.87
Probability	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078

The probability values given in the table are represented by the shaded area in the following diagram.



Using this table, above what weight would the heaviest 20% of bags of potatoes weigh?

Sample answer:

$$P(x < z) = 0.8 \Rightarrow z = 0.84$$

$$x = \mu + z\sigma$$

$$x = 2.05 + 0.84 \times 0.05$$

$$= 2.092 \text{ kg}$$

Criteria	**Marks**
Provides correct solution	**2**
Identifies the correct probability from the table, or equivalent merit	**1**
Markers comments	
Solution using 0.85 was also accepted. Many students used a z-score of 0.8 to mean 80%. Please be mindful of the difference between a z-score and a probability.	

Question 20 (5 marks)

Sixty people visited a swimming pool one afternoon. Thirteen out of the twenty-three people who wore goggles were adults. There were nineteen children.

a) Complete the two-way table below, showing this information.

Sample answer:

	Wore goggles	No goggles	Total
Adults	13	28	41
Children	10	9	19
Total	23	37	60

Criteria	Marks
Correctly completes the table	**2**
Completes the table with up to three errors	**1**
Markers comments	
Table was generally completed well.	

b) If an adult is selected at random, what is the probability they were wearing goggles?

Sample answer:

$$\frac{13}{41}$$

Criteria	Marks
Provides correct solution	**1**
Markers comments	
Generally completed well.	

c) An additional group of adults, not wearing goggles, arrived at the pool.
The probability that a person selected at random is an adult who is not wearing goggles is now 0.6.

How many additional adults arrived?

Sample answer:

Let x be the number of additional adults.

$$P(\text{adult not wearing goggles}) = \frac{28+x}{60+x}$$

$$\frac{3}{5} = \frac{28+x}{60+x}$$

$$3(60+x) = 5(28+x)$$

$$180+3x = 140+5x$$

$$40 = 2x$$

$$x = 20$$

Criteria	Marks
Provides correct solution	**2**
Finds an expression for *P*(adult not wearing goggles), or equivalent merit	**1**

Markers comments
Students who could write the correct expression generally could then find the answer. Lots of students started this question with a conditional probability mindset which led to issues. Checking solutions at the end would have made many students realise their solutions were not solutions. If in doubt, a guess a check method for this question was very do-able!

Question 21 (8 marks)

The derivative of a function $y = f(x)$ is given by $f'(x) = 3x^2 - 4x + 1$

a) Find the x-coordinates of the stationary points of $y = f(x)$ and determine the nature of the stationary points.

Sample answer:

For stationary points, $f'(x) = 0$.

$$3x^2 - 4x + 1 = 0$$
$$(3x-1)(x-1) = 0$$
$$x = \frac{1}{3}, 1$$

For the nature of the stationary points, consider the sign of $f''(x)$.

$$f''(x) = 6x - 4$$

$$f''\left(\frac{1}{3}\right) = 6\left(\frac{1}{3}\right) - 4$$
$$= -2 < 0 \quad \Rightarrow \text{ maximum turning point at } x = \frac{1}{3}$$

$$f''(1) = 6(1) - 4$$
$$= 2 > 0 \quad \Rightarrow \text{ minimum turning point at } x = 1$$

Criteria	Marks
Provides correct solution	**3**
Provides correct x coordinates AND correct expression for $f''(x)$, or equivalent merit	**2**
Provides correct x coordinates OR correct expression for $f''(x)$, or equivalent merit	**1**

Markers comments

Students generally found x-values of stationary points well.

Issues generally occurred when naming the nature of stationary points or finding the y-intercepts in this question through integration. This part of this question did not require any integration!

b) The curve $y = f(x)$ has a y-intercept at $y = -2$.

Find an expression for $f(x)$.

Sample answer:

$$f(x) = \int 3x^2 - 4x + 1 \, dx$$
$$= x^3 - 2x^2 + x + c$$
$$f(0) = -2 \Rightarrow c = -2$$
$$f(x) = x^3 - 2x^2 + x - 2$$

Criteria	Marks
Provides correct solution	**2**
Provides a correct primitive without evaluating c, or equivalent merit	**1**
Markers comments	
Students generally completed this part well.	

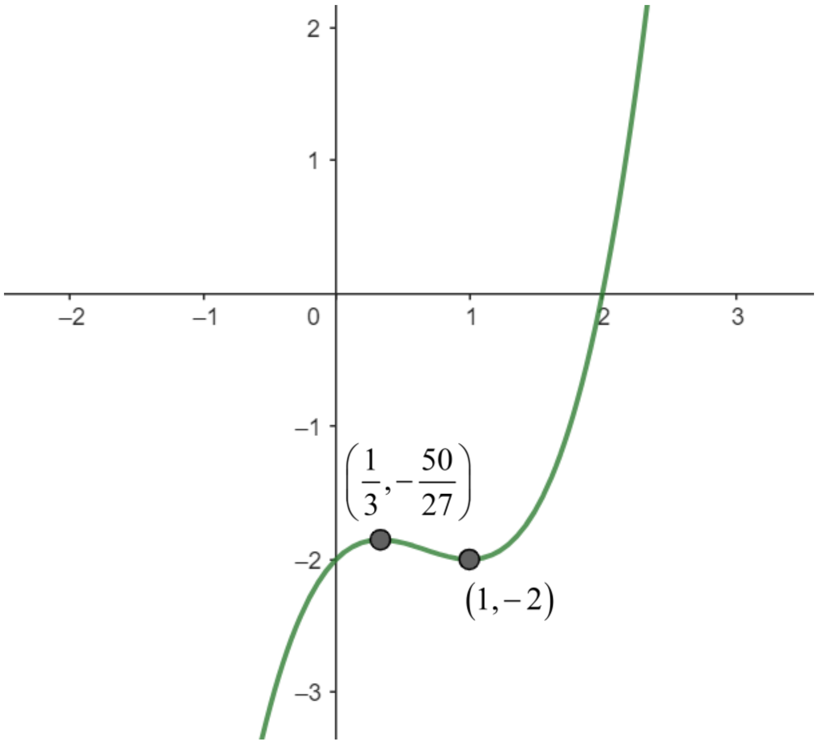
c) Sketch the curve, clearly indicating the stationary points and intercepts on the axes.

Sample answer:

$$\begin{aligned} f(x) &= x^3 - 2x^2 + x - 2 \\ &= x^2(x-2) + 1(x-2) \\ &= (x^2+1)(x-2) \end{aligned}$$

$$\begin{aligned} f\left(\frac{1}{3}\right) &= \left(\frac{1}{3}\right)^3 - 2\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right) - 2 \\ &= -\frac{50}{27} \end{aligned}$$

$$\begin{aligned} f(1) &= (1)^3 - 2(1)^2 + (1) - 2 \\ &= -2 \end{aligned}$$



Criteria	Marks
Provides correct graph with points indicated as required	**2**
Provides a graph with some correct features, or equivalent merit	**1**

Markers comments
Essentially all students integrated into a cubic function so there was no excuse to draw a graph that did not look like a cubic. Cubic graphs with a point correct were awarded a mark. Graphs that were distinctly not cubic required at least 3 of the 4 key points correct.

d) For what values of x is the curve concave down?

Sample answer:

Curve is concave down when $f''(x) < 0$.

$$
\begin{aligned}
6x - 4 &< 0 \\
6x &< 4 \\
x &< \frac{2}{3}
\end{aligned}
$$

Criteria	Marks
Provides correct solution	**1**

Markers comments
Students who realised the need for the second derivative generally did this question well.

Question 22 (3 marks)

A and B are two events such that $P(A)=0.3$, $P(B)=0.5$ and $P(A|B)=0.4$.

a) Show that $P(A\cup B)=0.6$

Sample answer:

$$\begin{aligned}P(A\cap B)&=P(A|B)\times P(B)\\&=0.4\times 0.5\\&=0.2\end{aligned}$$

$$\begin{aligned}P(A\cup B)&=P(A)+P(B)-P(A\cap B)\\&=0.3+0.5-0.2\\&=0.6\end{aligned}$$

Criteria	Marks
Provides correct solution	**2**
Attempts to use conditional probability formula, or equivalent merit	**1**
Markers comments	
Many responses generated an answer of 0.6 by performing various operations with the given probabilities, but these calculations were mostly invalid. Responses must have utilised the conditional probability formula to determine $P(A\cap B)$ to be awarded marks.	

b) Find $P(\text{neither } A \text{ nor } B)$

Sample answer:

$$\begin{aligned} P(\text{neither } A \text{ nor } B) &= 1 - P(A \cup B) \\ &= 1 - 0.6 \\ &= 0.4 \end{aligned}$$

Criteria	Marks
Provides correct solution	**1**
Markers comments	
Students are encouraged to construct a Venn diagram to visualise how this problem relates to their previous answer.	

Question 23 (3 marks)

Find two possible equations of a semi-circle with range $[3,5]$, given that it is an even function.

Sample answer:

Radius = 2

Option 1:

Centre at (0, 3), concave down

i.e. top half of circle

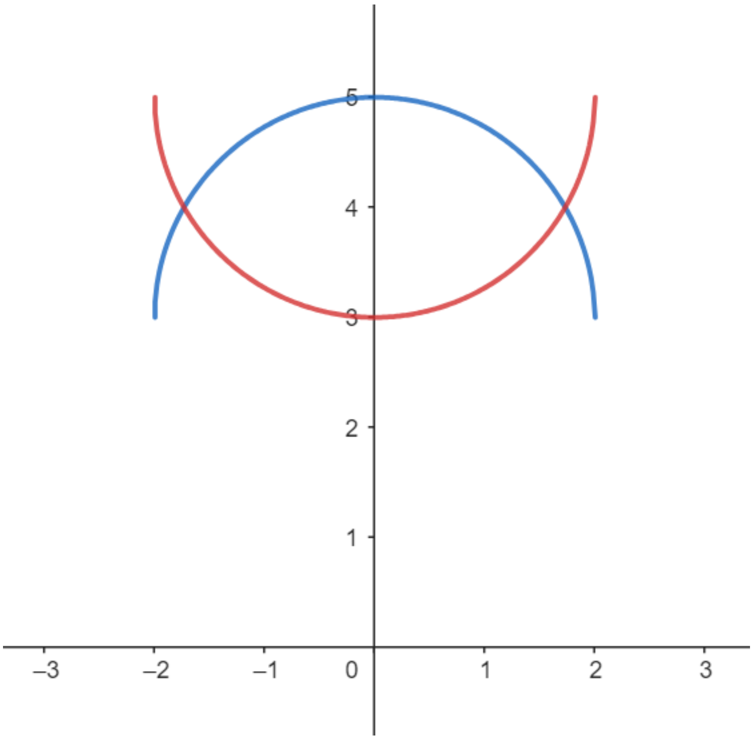
$y = 3 + \sqrt{4 - x^2}$

Option 2:

Centre at (0, 5), concave up

i.e. bottom half of circle

$y = 5 - \sqrt{4 - x^2}$



Criteria	Marks
Provides two correct equations	**3**
Provides one correct equation, or equivalent merit	**2**
Finds the radius or centre, or equivalent merit	**1**
Markers comments	
Few responses were awarded marks in this question. Better responses drew a diagram that assisted with determining the correct centre and radius. Areas for improvement include graphs of even functions, the equation of a circle, the equation of a semi-circle.	

Question 24 (2 marks)

A lift accelerates from rest at a constant rate until it reaches a speed of 5 metres per second. It continues at this speed for 12 seconds and then decelerates at a constant rate before coming to rest. The total travel time for the lift is 30 seconds.

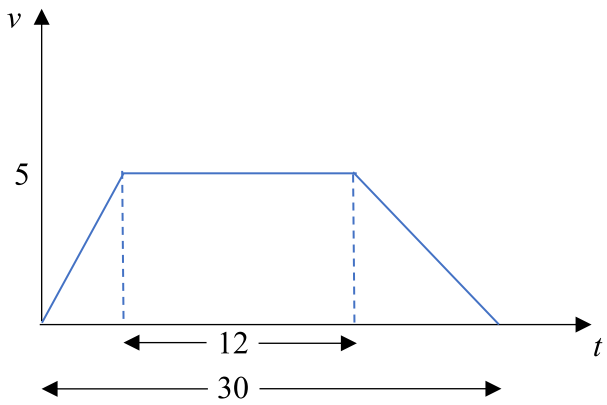
Find the total distance, in metres, travelled by the lift.

Sample answer:

Distance travelled = area of trapezium

$$= \frac{5}{2}(12+30)$$

$$= 105 \text{ metres}$$



Criteria	Marks
Provides correct solution	**2**
Provides a graph of *v* against *t*, determining the distance travelled during the period of constant speed, or equivalent merit	**1**
Markers comments	
Few students were awarded full marks for this question. Students would benefit from drawing a diagram to represent the problem rather than attempting to apply physics formulae. Students should be reminded that this course is about applying calculus to practical problems rather than applying formulae taught in another subject. Finding the area of a trapezium was problematic for many students.	

Question 25 (3 marks)

The table shows the probability distribution of a discrete random variable.

x	0	1	2	3	4
$P(X=x)$	p^2+p	p^2	p	$2p^2$	p

a) Show that $p=\frac{1}{4}$.

Sample answer:

$$p^2+p+p^2+p+2p^2+p=1$$
$$4p^2+3p-1=0$$
$$(4p-1)(p+1)=0$$
$$p=\frac{1}{4} \text{ or } -1$$

But $p\geq 0$, thus $p=\frac{1}{4}$

Criteria	Marks
Provides correct solution	**2**
Provides correct simplified equation involving *p*	**1**

Markers comments
Some responses included a correct equation but then factorised incorrectly or applied the quadratic formula incorrectly.

b) Find the expected value $E(X)$

Sample answer:

$$E(X)=\sum x\cdot p(x)$$
$$=0\left(\frac{5}{16}\right)+1\left(\frac{1}{16}\right)+2\left(\frac{1}{4}\right)+3\left(2\times\frac{1}{16}\right)+4\left(\frac{1}{4}\right)$$
$$=\frac{31}{16}$$

Criteria	Marks
Provides correct solution	**1**
Markers comments	
Students who attempted this question tended to be successful. Some responses had incorrect answers but were awarded the mark as they had shown the correct substitution line.	

Question 26 (3 marks)

Show that $\log_e(1+\sin x)-\log_e(1-\sin x)=2\log_e(\sec x+\tan x)$

Sample answer:

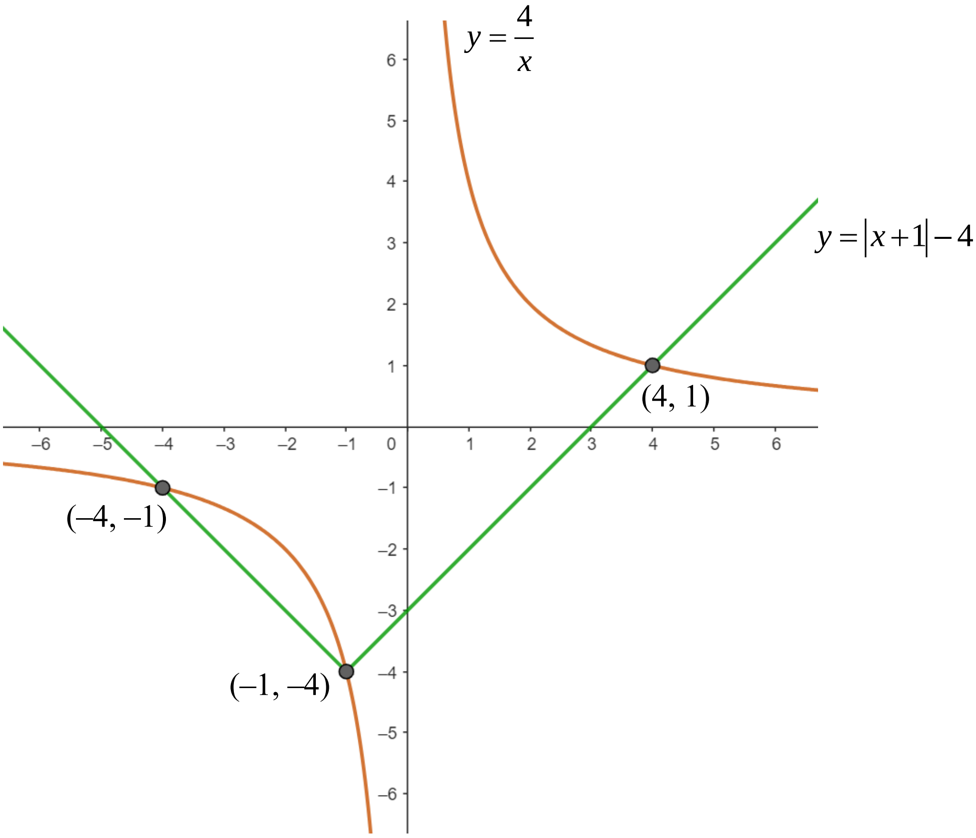
$$
\begin{aligned}
2\log_e\left(\sec x+\tan x\right) &= \log_e\left(\sec x+\tan x\right)^2 \\
&= \log_e\left(\frac{1}{\cos x}+\frac{\sin x}{\cos x}\right)^2 \\
&= \log_e\left(\frac{\left(1+\sin x\right)^2}{\cos^2 x}\right) \\
&= \log_e\left(\frac{\left(1+\sin x\right)^2}{1-\sin^2 x}\right) \\
&= \log_e\left(\frac{\left(1+\sin x\right)^2}{\left(1+\sin x\right)\left(1-\sin x\right)}\right) \\
&= \log_e\left(\frac{1+\sin x}{1-\sin x}\right) \\
&= \log_e\left(1+\sin x\right)-\log_e\left(1-\sin x\right)
\end{aligned}
$$

Criteria	Marks
Provides correct solution	**3**
Makes significant progress towards solution	**2**
Attempts to use some correct logarithm laws or trigonometric identities	**1**
Markers comments	
Few students achieved full marks for this question. Students should review log laws and perfect square expansions and trig identities. Many students appeared to have some valid approaches, but their execution contained multiple errors.	

Question 27 (6 marks)

a) Sketch the graphs of the functions $y=\frac{4}{x}$ and $y=|x+1|-4$ on the number plane below. Clearly show their point(s) of intersection and any intercepts on the axes.

Sample answer:



Criteria	Marks
Provides two correct graphs with points indicated as required	**4**
Provides two correct graphs with no points of intersection indicated, or equivalent merit	**3**
Provides one correct graph, or equivalent merit	**2**
Shows some progress towards one correct graph, or equivalent merit	**1**

Markers comments
Some students successfully graphed both functions, taking care to clearly indicate both the shape of the graphs and the points of intersection. Since hyperbolas were first taught in year 10, it should have been an easier graph to sketch.

b) Hence, or otherwise, solve $|x+1|-4 \leq \frac{4}{x}$

Sample answer:

$[-4,-1] \cup (0,4]$

Criteria	Marks
Provides correct solution	**2**
Provides one correct solution interval, or equivalent merit	**1**

Markers comments
Most students tried to solve algebraically, making many errors. Use of the word hence should have indicated that the students were to use the graph drawn in part a. Better responses knew they needed where the absolute value graph was below the hyperbola.

Question 28 (5 marks)

Consider $f(x)=\sqrt{1-x^2}$

a) Use the trapezoidal rule with 4 subintervals to estimate $\int_0^1 \sqrt{1-x^2}\,dx$.

Sample answer:

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$f(x)$	1	$\sqrt{\frac{15}{16}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{7}{16}}$	0

$$\int_0^1 \sqrt{1-x^2}\,dx \approx \frac{1}{8}\left[1+0+2\left(\sqrt{\frac{15}{16}}+\sqrt{\frac{3}{4}}+\sqrt{\frac{7}{16}}\right)\right]$$
$$\approx 0.748927\ldots$$

Criteria	Marks
Provides correct solution	**2**
Finds correct function values, or equivalent merit	**1**

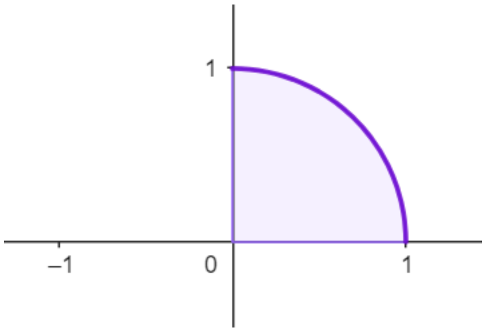
Markers comments

Was not well done for a straight Trapezoidal Rule. There was some confusion between 4 sub-intervals and 4 function values. Students need to be careful when they substitute in values as some missed the squaring part. Where possible try not to convert exact values to decimals.

b) Find the exact value of $\int_0^1 \sqrt{1-x^2}\,dx$ and hence find an approximation for π correct to 3 decimal places. **2**

Sample answer:

$$\int_0^1 \sqrt{1-x^2}\,dx = \text{area of quadrant}$$
$$= \frac{1}{4} \times \pi \times 1^2$$
$$= \frac{\pi}{4}$$



$$\frac{\pi}{4} \approx 0.748927...$$
$$\pi \approx 2.99570...$$
$$\approx 2.996$$

Criteria	Marks
Provides correct solution	**2**
Provides correct exact value of the definite integral, or equivalent merit	**1**
Markers comments	
Most students did not realise that the curve formed a quarter of a circle so they could use the area formula from year 8. Then match the quadrant are to their answer from part a. Students need to realise that when they see the word hence, it means to use the previous part in your working for this part.	

c) Explain why this value under-estimates the exact value of π .

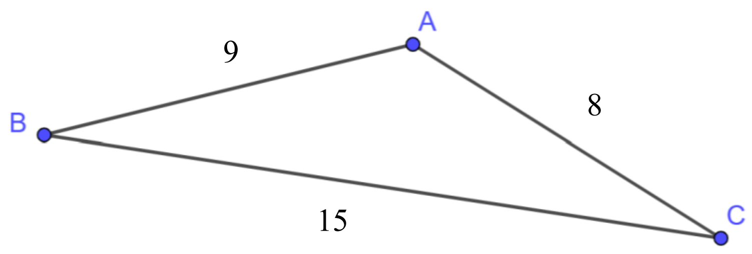
Sample answer:

The curve is concave down, so all trapeziums used to estimate its area sit entirely below the curve.

Criteria	Marks
Provides correct reason	**1**
Markers comments	
Better responses referred to the Quarter circle being concave down. Drawing diagrams also helped support your explanation.	

Question 29 (4 marks)

The diagram shows a triangle *ABC* where $AB = 9$, $AC = 8$ and $BC = 15$. **4**



Find the area of the triangle *ABC*. Give your answer correct to one decimal place.

Sample answer:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$= \frac{15^2 + 8^2 - 9^2}{2 \times 15 \times 8}$$
$$= \frac{13}{15}$$
$$C = \cos^{-1}\left(\frac{13}{15}\right)$$
$$= 29^\circ 56'$$

$$\text{Area} = \frac{1}{2} ab \sin C$$
$$= \frac{1}{2} \times 8 \times 15 \times \sin 29^\circ 56'$$
$$= 29.9332\ldots$$
$$= 29.93 \text{ units}^2$$

Criteria	Marks
Provides correct solution	**4**
Correctly finds one angle and makes some progress towards finding area, or equivalent merit	**3**
Correctly finds one angle in the triangle, or equivalent merit	**2**
Attempts to use cosine rule or area of a triangle formula, or equivalent merit	**1**

Markers comments
Most students did well on this question, using either Cosine Rule or Sine Rule to find one of the angles that could then be applied to the Area of a Triangle formula.

Question 30 (4 marks)

a) Show that $\dfrac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} \equiv \sec^2 x$.

Sample answer:

$$\begin{aligned}\frac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} &= \frac{\dfrac{1}{\sin x}}{\dfrac{1}{\sin x} - \sin x} \\ &= \frac{1}{1 - \sin^2 x} \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x\end{aligned}$$

Criteria	Marks
Provides correct solution	**2**
Finds the expression in terms of $\sin x$, or equivalent merit	**1**

Markers comments

Most students were able to express the LHS in terms of $\sin x$. However, some students then had trouble simplifying the resulting compound fraction. Students who used methods other than expressing in terms of $\sin x$ had difficulties achieving the required result.

b) Hence, or otherwise, find the exact value of $\int_0^{\frac{\pi}{3}} \tan^2 x \left(\frac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} \right) dx$

Sample answer:

$$\begin{aligned}
\int_0^{\frac{\pi}{3}} \tan^2 x \left(\frac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} \right) dx &= \int_0^{\frac{\pi}{3}} \tan^2 x \sec^2 x \, dx \\
&= \left[\frac{1}{3} \tan^3 x \right]_0^{\frac{\pi}{3}} \\
&= \frac{1}{3} \left[\tan^3 \left(\frac{\pi}{3} \right) - \tan^3 (0) \right] \\
&= \frac{1}{3} \left[\left(\sqrt{3} \right)^3 - 0 \right] \\
&= \frac{1}{3} \left(3\sqrt{3} \right) \\
&= \sqrt{3}
\end{aligned}$$

Criteria	Marks
Provides correct solution	**2**
Recognises the integral is of the form $\int f'(x)\left[f(x) \right]^n dx$, or equivalent merit	**1**

Markers comments
This was poorly done; very few students recognised the integral was of the form $\int f'(x)\left[f(x) \right]^n dx$. Without this step, this question was unachievable at this level. There were many, many invalid integrals given as answers to this question. If an incorrect indefinite integral was found, but a student demonstrated the correct steps to substitute the x values and find the correct exact value for their integral, they were awarded 1 mark.

Question 31 (1 mark)

For a given discrete probability distribution, $E(X) = 5$ and $\text{Var}(X) = 3$.

Find the value of $E(2X + 1)$.

Sample answer:

$$\begin{aligned} E(2X+1) &= 2\times 5+1 \\ &= 11 \end{aligned}$$

Criteria	Marks
Provides correct solution	**1**
Markers comments	
Mixed results on this question. Many students tried to evaluate complicated expressions involving variance.	

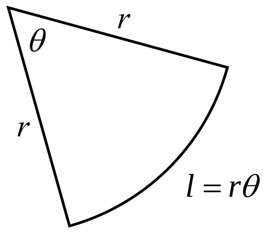
Question 32 (5 marks)

A wire of length L is bent to form a sector with radius r and angle θ.

a) Show that $\theta = \dfrac{L-2r}{r}$.

Sample answer:

$$L = 2r + r\theta$$
$$r\theta = L - 2r$$
$$\theta = \frac{L-2r}{\theta}$$



Criteria	Marks
Provides correct solution	**2**
Provides an expression for L in terms of r and θ, or equivalent merit	**1**

Markers comments

Mostly done well. The best responses to this question included a diagram. Students should take care not to use L for arc length when it is already used for the total length of the wire in the question

b) Show that the area A of the sector is maximised when $r = \frac{L}{4}$.

Sample answer:

$$
\begin{aligned}
A &= \frac{1}{2}r^2\theta \\
&= \frac{1}{2}r^2\left(\frac{L-2r}{r}\right) \\
&= \frac{Lr}{2} - r^2
\end{aligned}
$$

For maximum area, need $\frac{dA}{dr} = 0$ and $\frac{d^2A}{dr^2} < 0$

$$
\begin{aligned}
\frac{dA}{dr} &= \frac{L}{2} - 2r \\
0 &= \frac{L}{2} - 2r \\
2r &= \frac{L}{2} \\
r &= \frac{L}{4}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d^2A}{dr^2} &= -2 \\
&< 0
\end{aligned}
$$

Thus area is maximised when $r = \frac{L}{4}$

Criteria	Marks
Provides correct solution	**3**
Correctly shows there is a stationary point at $r = \frac{L}{4}$ but fails to show it is a maximum, or equivalent merit	**2**
Provides a correct expression for A in terms of r and L, or equivalent merit	**1**

Markers comments

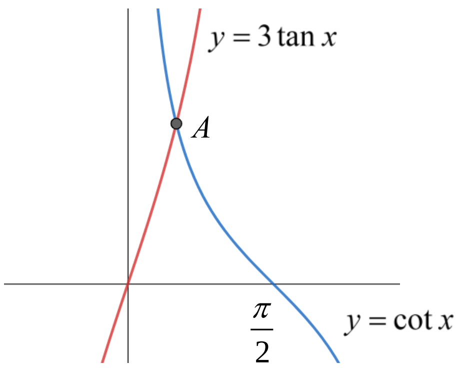
Most students were able to successfully substitute the expression for θ found in part (a) into the formula for the area of a sector to gain one mark. However, several students failed to simplify this expression before differentiating. This resulted in needing to use the product and/or/quotient rule for differentiation, where many mistakes were made. Some students also left the area of a sector formula in degrees instead of radians, resulting in an incorrect expression for A.

Students should remember that when solving a maxima/minima problem, it is vital they determine the nature of the stationary point to show whether it is a maximum or minimum value that has been found.

Question 33 (5 marks)

Part of each of the curves $y = 3\tan x$ and $y = \cot x$ are shown on the graph below.

The curve $y = \cot x$ cuts the x axis at $x = \frac{\pi}{2}$ and the two curves intersect at the point A.



a) Show that the x coordinate of A is $x = \frac{\pi}{6}$.

Sample answer:

$$3\tan x = \cot x$$
$$3\tan^2 x = 1$$
$$\tan^2 x = \frac{1}{3}$$
$$\tan x = \pm\frac{1}{\sqrt{3}}$$
$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \ldots$$

But at A, $x < \frac{\pi}{2}$, thus $x = \frac{\pi}{6}$

Criteria	Marks
Provides correct solution	**2**
Provides a correct equation involving $\tan^2 x$, or equivalent merit	**1**
Markers comments	
Most students attempted to solve the two equations simultaneously, expressing everything in terms of either tan *x*, sin *x*, or cos *x* to form a quadratic. However, very few students recognised the need to consider both the positive and negative square root. In this question, it was known that the solution was between 0 and $\frac{\pi}{2}$, so you were given the benefit of the doubt and still awarded full marks if you only found the solution for *x* in the first quadrant. Better responses showed that there are solutions to the quadratic equation in all four quadrants, but then stated that the particular solution needed here was in the first quadrant. If this question was simply asking you to solve the trig equation, very few students would have achieved full marks.	

b) By considering $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$, find the exact area enclosed by the curves and the x axis between $x = 0$ and $x = \frac{\pi}{2}$.

Sample answer:

$$\text{Area} = \int_0^{\frac{\pi}{6}} 3\tan x\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot x\,dx$$

$$= 3\int_0^{\frac{\pi}{6}} \frac{\sin x}{\cos x}dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}dx$$

$$= -3\int_0^{\frac{\pi}{6}} \frac{-\sin x}{\cos x}dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x}dx$$

$$= -3\left[\ln|\cos x|\right]_0^{\frac{\pi}{6}} + \left[\ln|\sin x|\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= -3\left[\ln\left(\cos\frac{\pi}{6}\right) - \ln(\cos 0)\right] + \left[\ln\left(\sin\frac{\pi}{2}\right) - \ln\left(\sin\frac{\pi}{6}\right)\right]$$

$$= -3\left[\ln\left(\frac{\sqrt{3}}{2}\right) - \ln(1)\right] + \left[\ln(1) - \ln\left(\frac{1}{2}\right)\right]$$

$$= -3\ln\left(\frac{\sqrt{3}}{2}\right) - \ln\left(\frac{1}{2}\right)$$

$$= \ln\left(\frac{8}{3\sqrt{3}}\right) + \ln 2$$

$$= \ln\left(\frac{16}{3\sqrt{3}}\right)$$

Criteria	Marks
Provides correct solution	**3**
Provides an antiderivative, or equivalent merit	**2**
Provides an integral expression for the area, or equivalent merit	**1**

Markers comments

Most students were able to identify the correct integral needed for the required area. However, few students received full marks in this question. Common errors included:

- ignoring the hint to consider $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$ and instead incorrectly ending up with anti-derivatives that involved $\sec^2 x$ or other trig functions, without a logarithm
- multiplying the wrong term by -1 ($\cot x$ instead of $\tan x$) in order to integrate and get a logarithm
- incorrect bounds on the integral or incorrect exact trig values, resulting in an answer involving ln(0), which is undefined

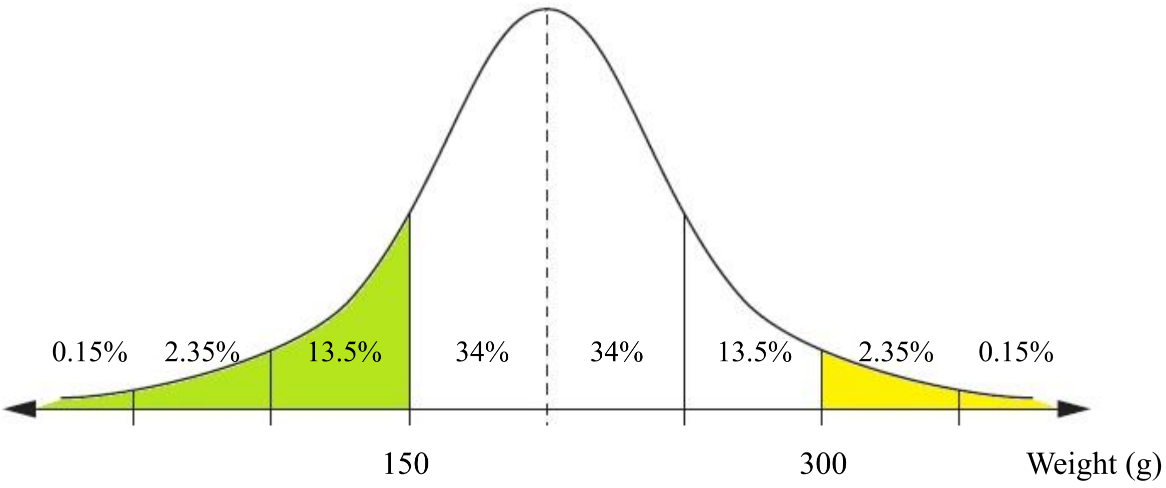
If a student found a correct expression for the area involving logs, but then incorrectly applied log laws in an attempt to simplify this expression, they were not penalised. ISE (ignore subsequent error) is written on the paper in these instances. Students were not expected to simplify their answer in order to receive 3 marks.

Question 34 (2 marks)

A farming company harvests oranges to sell to grocery stores. Oranges that weigh less than 150 grams are rejected and used for juicing instead. Typically, 16% of all oranges are rejected. Also, oranges that are larger than 300 grams are reserved for restaurants. Typically, 2.5% of all oranges are sold to restaurants.

If the weights of the oranges are normally distributed, find the mean and standard deviation of the normal distribution.

Sample answer:



Standard deviation $= (300 - 150) \div 3$

$= 50\,\text{g}$

Mean $= 150 + 50$

$= 200\,\text{g}$

Criteria	Marks
Provides correct solution	**2**
Makes some progress towards finding a correct solution	**1**
Markers comments	
Most students who attempted this question answered it correctly. Better responses drew a diagram like in the above sample answer and from there it was easy to calculate the missing values. Another successful method used by many students was to identify that a score of 150 had a z score of -1 and a score of 300 had a z score of 2, and then substitute these values into the z score formula and solve the two equations simultaneously.	

Question 35 (8 marks)

A continuous random variable X has the probability density function $f(x)$ given by

$$f(x) = \begin{cases} k(\sin(\pi x) + 2x), & \text{for } 0 \le x \le 3 \\ 0, & \text{for all other values of } x \end{cases}$$

a) Show that $k = \dfrac{\pi}{2 + 9\pi}$.

Sample answer:

$$\int_0^3 k\left(\sin(\pi x) + 2x\right)dx = k\left[-\frac{1}{\pi}\cos(\pi x) + x^2\right]_0^3$$

$$1 = k\left[-\frac{1}{\pi}\cos(3\pi) + 3^2 - \left(-\frac{1}{\pi}\cos(0) + 0^2\right)\right]$$

$$1 = k\left[-\frac{1}{\pi}(-1) + 9 + \frac{1}{\pi}(1) - 0\right]$$

$$1 = k\left(\frac{2}{\pi} + 9\right)$$

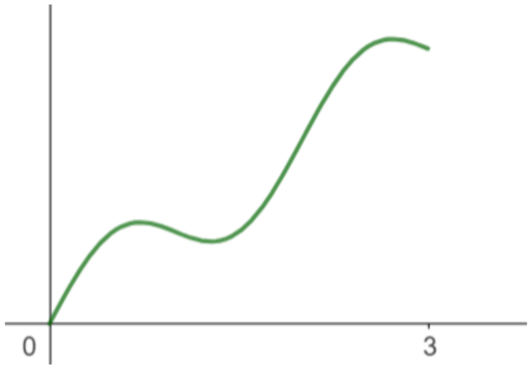
$$1 = k\left(\frac{2 + 9\pi}{\pi}\right)$$

$$k = \frac{\pi}{2 + 9\pi}$$

Criteria	Marks
Provides correct solution	**3**
Provides a correct equation involving a definite integral equal to 1 AND finds the antiderivative, or equivalent merit	**2**
Provides a correct equation involving a definite integral equal to 1 OR finds an antiderivative, or equivalent merit	**1**

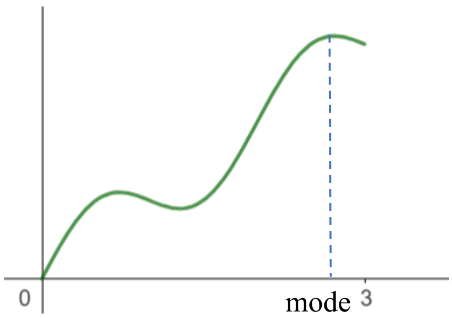
Markers comments
Most students correctly identified they needed to integrate the given PDF and solve it equal to zero, which scored them 1 mark. From here, many mistakes were made. Common errors included: • Integrating $\sin(\pi x)$ to give $\pi\cos(\pi x)$ or $-\pi\cos(\pi x)$, instead of $-\frac{1}{\pi}\cos(\pi x)$ • Not multiplying the k by the $2x$ • Not finding the exact value of $\cos(3\pi)$ Students should also avoid "fudging" their final line of working to give the answer expected in the question. If only one earlier error was made but all other steps were correct, resulting in an incorrect value of k, students could still receive 2 marks out of 3 for this question due to carry over error. However, if an incorrect answer miraculously turns into a correct answer by some invalid algebra, this cost students their carry over error mark.

b) The graph of the density function is given below.



Without any calculation, explain why the mode must be greater than the median.

Sample answer:



The mode occurs at the maximum value on the graph and the median occurs at the point which divides the area under the curve in half.

From the graph we can see the area to the left of the mode is clearly greater than half the total area. Thus, the median must be to the left of the mode i.e. mode > median.

Criteria	Marks
Provides a correct reason	**1**
Markers comments	
This question was terribly done. Most responses referenced the median being the middle score, with many students indicating this occurred at 1.5. In order to score a mark, students need to identify that the median occurs at the x value which divides the *area below the curve* in half.	

c) Find the mode of X. Give your answer correct to two decimal places.

Sample answer:

Mode occurs at a maximum turning point. For stationary points, $f'(x) = 0$.

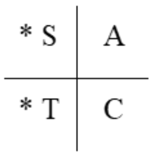
$$f(x) = \frac{\pi}{2+9\pi}(\sin(\pi x) + 2x)$$

$$f'(x) = \frac{\pi}{2+9\pi}(\pi\cos(\pi x) + 2)$$

$$0 = \pi\cos(\pi x) + 2$$

$$\cos(\pi x) = -\frac{2}{\pi}$$

$$\pi x = \cos^{-1}\left(-\frac{2}{\pi}\right)$$



$$\text{Reference } \angle = \cos^{-1}\left(-\frac{2}{\pi}\right)$$

$$= 0.88068...$$

$$\pi x = \pi - 0.88,\ \pi + 0.88,\ 3\pi - 0.88,\ 3\pi + 0.88,\ ...$$

$$= 2.2609,\ 4.0222,\ 8.5440,\ 10.3054,\ ...$$

$$x = 0.72,\ 1.28,\ 2.72,\ 3.28,\ ...$$

But $0 \le x \le 3$, and from the graph the mode is the third stationary point.

$\therefore$ mode is $x = 2.72$

Criteria	Marks
Provides correct solution	**4**
Finds x value for at least one stationary point, or equivalent merit	**3**
Provides an equation involving $\cos(\pi x)$, or equivalent merit	**2**
Attempts to find the derivative, or equivalent merit	**1**
Markers comments	
Very few students scored full marks for this question. Most students attempted to find $f'(x)$, however many errors were made. In particular, lots of responses missed that the $2x$ should also have been multiplied by the value for k. Many students were able to find one stationary point, but didn't consider the multiple solutions to a trigonometric equation.	